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New tools for fermion masses from extra dimensions

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Abstract: We present models in which the observed fermion masses and mixings are generated by dynamically localizing the three generations of matter in a flat compact extra dimension. We first construct models assuming the hierarchy problem is addressed by the existence of large extra dimensions, i.e. the fundamental scale is not far above a TeV and supersymmetry is not imposed. These models are compactified, chiral, and don't require fine-tuning to generate the top mass. Limits on the compactification scale based on flavor-changing neutral currents are relaxed compared to those on existing models. We then supersymmetrize some of these models. Using $\mathcal{N} = 1$ superspace language in extra dimensions, we find space-dependent flat directions which can be used to localize fields. Finally, we discuss methods of breaking supersymmetry and the impact of these models on the superpartner spectrum.

Keywords: Extra Large Dimensions, Supersymmetry Breaking, Supersymmetric Standard Model.

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1. Introduction

Measured fermion masses represent a window into ultraviolet physics. The standard model (SM) can reproduce the observed hierarchy of masses and mixing angles with a set of dimensionless parameters (Yukawa couplings) ranging over five orders of magnitude, but does not explain why such a diverse and interesting pattern exists.

One avenue of exploration of high energy physics would be to find models in which such hierarchical patterns can be reproduced by a theory with only "natural" couplings, i.e. dimensionless parameters of order unity. The first success of this type is the Froggatt-Nielsen mechanism [1] which imposes an additional symmetry on the SM thereby forbidding most Yukawa couplings. Yukawa couplings are generated by higher dimension operators and the spontaneous breaking of the additional symmetry and are suppressed by powers of the breaking scale over some fundamental scale.

An interesting orthogonal approach to generating a large hierarchy in the Yukawas is to use locality rather than symmetries to produce small dimensionless numbers. The Arkani-Hamed-Schmaltz (AS) mechanism requires SM fermion zero modes to be localized at different positions in one (or more) extra dimension(s) [2]. This can be done by coupling five-dimensional fermions to a scalar field with a space-dependent vacuum expectation value (VEV). For gaussian wave functions, couplings between fields are exponentially suppressed for separations of order a few (in units of the wave function widths). It has been shown [3] that all fermion masses and mixing angles can be reproduced by localizing all SU(2) doublets and SU(2) singlets at different positions in one extra dimension, with the Higgs zero mode constant along the extra dimension. However, it has been noted that in the five-dimensional case one cannot accommodate the observed CP violation in the Kaon system [4]. In addition the large top mass requires some fine-tuning of parameters. Many interesting variations on this theme have since appeared in the literature [5]–[8].

A complete version of a model of this type should have a four-dimensional chiral low energy effective theory. Five-dimensional theories are in general non-chiral but can be made chiral by choosing the right boundary conditions [9]. In the next section, we present a simple set of orbifold boundary conditions which can reproduce the AS model in a compact extra dimension. The boundary conditions are realized by compactifying on a $\mathcal{Z}_2$ orbifold and by giving each fermion a different 5d mass which is odd under the $\mathcal{Z}_2$, we localize each fermion at a distinct location in the extra dimension.

We then present simpler models in which the fermions (and in one of the models, the Higgs boson VEV) are each localized on one of two orbifold fixed points. Different Yukawa couplings are generated due to the fact that the fermion wave functions have different widths. Their widths are controlled by their order one couplings to a scalar field, and their location (i.e. which orbifold fixed point they are centered about) is governed by the sign of the coupling.[1] In this scenario (unlike the AS one), the top mass is natural and is a result of a quark doublet and a quark singlet having the opposite sign couplings as the other quarks, localizing them at the Higgs brane. In addition, ϵ_K is predicted to be of the right order because the Yukawa matrices are "full" in the sense that there are no negligible entries. Finally, the flavor problems common to these models are ameliorated and thus a lower compactification scale is allowed.

In section 3, we promote our models to a supersymmetric framework. We use the notation of Arkani-Hamed, Gregoire and Wacker [10] to describe supersymmetry in

[1] A supersymmetric model of this sort, in the case of an anti-de Sitter background, was discussed in [8].

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five dimensions. In the simplest model, zero modes are localized by mass terms which are odd under the orbifold. These terms are allowed by all remaining symmetries in the theory and can be viewed as VEV's of maximally broken gauge symmetries. We then find flat directions in which a scalar field has a space-dependent VEV along the extra dimension. This allows for additional models where chiral superfields are localized at arbitrary points. However, it is difficult to produce viable models of this type because of the extra constraints of 5d supersymmetry. These models can be made to work by supplementing them with a partial Froggatt-Nielsen mechanism. We also present another possibility where we compactify on S^1, and introduce chirality by hand by inserting a three brane and "projecting" the chiral states into the bulk using the supersymmetric profiles found earlier in the section.

In section 4 we discuss some of the issues which naturally come up in this context. For instance, are there any distinguishing signals from such models and how should supersymmetry be broken. For a high flavor scale, how supersymmetry breaks plays an important roll in determining whether or not one can find physical evidence of these theories. Mediating supersymmetry breaking can be done in an extra dimensional context, as in gaugino mediated supersymmetry breaking, or can be completely orthogonal to this flavor mechanism, such as low-scale gauge mediation.

2. Non-supersymmetric Models

Our first models use an extra dimension to explain the small Yukawa interactions apparent from the quark and lepton masses in terms of fermions localized in the extra dimension. Localizing quarks and leptons may also be helpful to prevent unacceptably fast proton decay [2, 11]. We assume a flat background metric, $\eta^{MN} = \mathrm{diag}[+1, -1, -1, -1, -1]$, where the large Latin characters $M, N, \ldots$ refer to the full 5d vector indices and space coordinates $x^M = \{x^\mu, y\}$ are decomposed into the 4d (uncompactified) subset x^μ and the compactified direction, y. Without supersymmetry, our models suffer from the usual hierarchy and triviality problems of the SM, and thus we would like the fundamental scale to be of order TeV so that the large extra dimension solution to the hierarchy problem [12, 13] is applicable.[2] As we wish to construct flavor by localizing the fermions of the SM at various points in the extra dimension, it is necessary that the SM gauge fields live in the full 5d theory. We begin by constructing models which reproduce the fermion mass spectrum. We then examine the effects of this new physics on low energy processes allowing us to put a bound on the size of the extra dimension and of the fundamental scale.

We work with a compact extra dimension subject to orbifold boundary conditions, $S^1/\mathcal{Z}_2$, with the orbifold fixed points at $y = 0$ and $y = \pm L/2$. The orbifold

[2]For example, we might have more than one extra dimension, with the SM fields seeing only one of the extra dimensions.

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is essential in order to recover a chiral theory from the vector-like 5d theory by removing the mirror partners of the fermion zero modes. It is further useful because it can force the VEV of an odd scalar field to assume a non-trivial profile with respect to the extra dimension. The extra dimension is compact, with $-L/2 < y \leq L/2$ and the points $-L/2$ and $L/2$ identified, but the orbifold constrains the fields in the region $y < 0$ to shadow the fields in the $y > 0$ region, and thus the physical dynamics may be understood to take place in the region $0 \leq y \leq L/2$. The 5d theory contains a real scalar "localizer" field ϕ and a number of fermions ψ (which correspond to the usual SM quarks and leptons plus their mirror partner degrees of freedom) satisfying orbifold boundary conditions [9],

$$\begin{aligned} \phi(x^\mu, -y) &= -\phi(x^\mu, y)\,, \qquad & \phi(x^\mu, L/2 + y) &= -\phi(x^\mu, -L/2 + y)\,, \\ \psi(x^\mu, -y) &= \gamma_5 \psi(x^\mu, y)\,, \qquad & \psi(x^\mu, L/2 + y) &= \gamma_5 \psi(x^\mu, -L/2 + y)\,. \end{aligned} \tag{2.1}$$

The 5d lagrangian density is,

$$\mathcal{L}_5 = \overline{\psi} \left[i\gamma^M \partial_M - \frac{f_\psi}{\sqrt{M_*}} \phi \right] \psi + \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{\lambda}{4M_*} (\phi^2 - u^2)^2\,, \tag{2.2}$$

where we have ignored the gauge interactions as they are unimportant with respect to localization. A mass for the fermions is forbidden by the orbifold transformations (2.1). The fundamental scale M_* has been included in the interactions so that the coupling constants f_ψ and λ are dimensionless.

In order to estimate reasonable ranges of the parameters such as f_ψ and λ, it is necessary to make some assumptions about the underlying theory. If the underlying theory at high energies is such that all couplings are strong at the cut-off, naive dimensional analysis (NDA) would suggest $f_\psi \sim \sqrt{24\pi^3}$ and $\lambda \sim 24\pi^3$ at M_* [14]. These estimates also provide an estimate of where perturbation theory is expected to break down. We will consider couplings somewhat smaller than those suggested by NDA, $f_\psi \sim \lambda \sim 1$. Such couplings could be considered natural for a perturbative underlying theory containing only one dimensional parameter M_* (see for example [15, 16]). In constructing models, we invoke sources for bulk fields living on branes (and in some cases field theories confined to the branes themselves). We will assume the underlying theory is such that the branes can be treated as thin, rigid objects, and further that the sources living on them (which we presume have some unspecified dynamical origin) are generated at a high scale, and will not succumb to back-reaction effects from the bulk fields.

As discussed in [17], the orbifold boundary conditions clash with the bulk dynamics for ϕ, resulting in a non-trivial VEV which can be approximated for $M_* L \gg \lambda$ by,

$$\langle \phi \rangle (y) = u \tanh\left[\beta(-L/2 - y)\right] \tanh\left[\beta y\right] \tanh\left[\beta(L/2 - y)\right]\,, \tag{2.3}$$

where $\beta^2 = \lambda u^2/2$. This nontrivial profile for $\langle \phi \rangle$, inserted into the 5d lagrangian 2.2, appears as a mass for fermion ψ that varies across the extra dimension, $M_\psi(y) =$

$f_\psi\langle\phi\rangle(y)$. Turning to the 4d effective theory, we expand ψ in a Kaluza-Klein (KK) tower [18] and find expressions for the zero mass wave functions,

$$\psi^0_\pm(y) = \mathcal{N}_\pm \mathrm{Exp}\left[f_\psi \int_0^y dy' \langle\phi\rangle(y')\right], \tag{2.4}$$

where ψ^0_+ is the left-chiral zero mode and ψ^0_- the right-chiral one. For $f_\psi u > 0$, this results in wave function ψ^0_+ localized about the orbifold fixed point $y = 0$, with profile that looks like $1/\cosh^{(\alpha/\beta)}[\beta y]$, where $\alpha = |f_\psi u|$. For $f_\psi u < 0$, ψ^0_+ has similar profile, but centered about the other orbifold point, $y = L/2$. In each case, the mirror zero mode ψ^0_- is inconsistent with the orbifold conditions, eq. (2.1), and thus removed from the spectrum [17].

In order to simplify our analysis, we will further consider the case in which $\lambda u^2 L^2 \gg 1$, for which the domain walls can be approximated as step functions,

$$\langle\phi\rangle(y) = u\,\epsilon(y) \tag{2.5}$$

with,

$$\epsilon(y) = \begin{cases} +1 & \frac{L}{2} > y > 0 \\ -1 & \frac{-L}{2} < y < 0 \end{cases} \tag{2.6}$$

with $L/2$ and $-L/2$ identified. Again we have a single zero mode with profile,

$$\psi^0(y) = \sqrt{\frac{2\alpha}{1 - e^{-\alpha L}}}\, e^{-\alpha y}, \tag{2.7}$$

centered at $y = 0$ for $f_\psi u > 0$, or the same wave function with by $y \to L/2 - y$ for $f_\psi u < 0$. In regards to flavor in the light quark sector our exponential profile does not actually differ much from $1/\cosh$ because the small entries in the Yukawa interaction matrices are generated by the small overlap in the tails of the exponentials which do not differ much from the tails of the $1/\cosh$ function. The major difference is with respect to the large mass matrix entries, namely the top Yukawa coupling, which come from large overlaps, and thus are more sensitive to the shape of the entire wave functions. In that case, as we shall see below, the more narrow exponential will have greater difficulty realizing an $\mathcal{O}(1)$ top Yukawa coupling than the broader $1/\cosh$ would have, and thus it is somewhat more difficult to realize flavor for our limiting case than it would be for the general $1/\cosh$ zero modes.

We now discuss the AS model, the original proposal to generate flavor in a large extra dimension [2], and construct two explicit non-supersymmetric models of flavor, finishing with some remarks on the experimental constraints on this class of models, and whether they allow the extra dimension to really solve the hierarchy problem.

2.1 The Arkani-Hamed-Schmaltz model

The AS model generates flavor by localizing zero modes of the weak doublet and singlet fermions at different positions, with the Higgs VEV spread evenly throughout the bulk. The 4d Yukawa interactions arise as the overlap of a doublet with a singlet field. It is assumed that the wave functions are gaussians with common widths α (presumably of the order of the fundamental scale M_*); flavor is successfully realized by distributing the fermions appropriately throughout a region of about $\Delta y \sim 25/\alpha$, determined from a numerical scan of parameters by Mirabelli and Schmaltz [3].

In order to remove the troublesome mirror fermions, it is desirable to impose the orbifold on the AS model. Since we need to have the fermion zero modes spread (roughly) evenly through the extra dimension, and to have gaussian wave functions, we would like the localizer VEV to be approximately linear. This can be engineered by including sources for $\partial_y \phi$ at the orbifold fixed points,

$$J_1 \left(\partial_y \phi\right) \delta\left(y\right) + J_2 \left(\partial_y \phi\right) \delta\left(y - L/2\right). \tag{2.8}$$

If ϕ were massless and had no quartic interaction, these sources would literally result in a linear VEV. For a massive localizer, the VEV may be simply obtained by using the Green's function for a simple harmonic oscillator of imaginary frequency and we find that when $m \lesssim M_*/3$ (as could be expected if m^2 is generated at one-loop), the VEV remains approximately linear, as demonstrated in figure 1. In order to have each fermion localized about a different point in the extra dimension, we introduce "odd masses" in $\mathcal{L}_5$ for each fermion,

$$\mathcal{L}_5^M = M_\psi \; \epsilon(y) \; \overline{\psi} \, \psi \, . \tag{2.9}$$

This term could come from the VEV of an second scalar field which is odd under the $\mathcal{Z}_\in$ orbifold symmetry and has the appropriate sources at the orbifold fixed points. The odd mass effectively shifts the zero crossing of the "mass function" for the fermion, thus localizing it some distance away from one of the orbifold fixed points.

2.2 Higgs in the bulk

In our first model, the Higgs lives in the entire 5d bulk, and is even under the orbifold transformation. A 5d version of the SM Higgs potential will thus generate an EWSB VEV v spread evenly throughout the extra dimension. The underlying 5d Yukawa interactions are,

$$\mathcal{L} = \frac{Y^d_{ij}}{\sqrt{M_*}} H \overline{q}_i d_j + \frac{Y^u_{ij}}{\sqrt{M_*}} H^c \overline{q}_i u_j + h.c. \;, \tag{2.10}$$

where H is the Higgs doublet, $H^c = i\sigma_2 H^*$, q_i with $i = 1, 2, 3$ are the three families of weak doublet quarks, and u_i and d_i are the up- and down-type weak singlet quarks, respectively. We have included the appropriate power of the cut-off scale for the

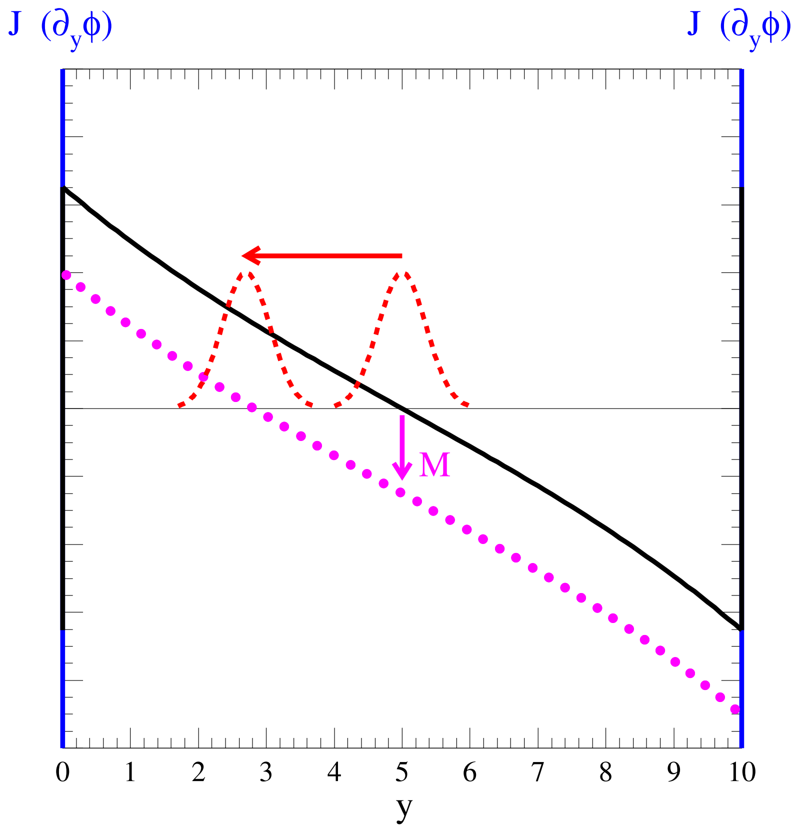

Figure 1: The profile (solid curve) for $\langle\phi\rangle$ resulting from the sources eq. 2.8 with $J_1 = J_2$ and the mass of the localizer taken to be $m \sim M_*/5$. Also shown is the effective mass function (dotted curve) seen by a fermion with odd mass $M \sim J$, and shift in the wave function (dashed curves) which results from this odd mass.

effective 5d theory, M_*, such that the Y^u and Y^d are dimensionless. Again, we assume that all of the Y^u and Y^d are of $\mathcal{O}(1)$ (though not necessarily identically equal to one, and with $\mathcal{O}(1)$ complex phases with respect to one another).

We realize a hierarchy in the effective 4d theory by coupling the weak doublets to ϕ with couplings $f_{q_i} > 0$ and the weak singlets to ϕ with $f_{u_i}, f_{d_i} < 0$. This results in the doublet zero modes centered at $y = 0$ with exponential widths α_i while the singlet zero modes (both up- and down-type) are centered at $y = L/2$, again with a variety of widths. Provided $u \sim M_*^{3/2}$, $\lambda \sim 1$, and $f_i \sim 1$, the widths α will be of order M_*. The effective coupling strength between the Higgs and the zero modes of the left-handed doublet i and right-handed singlet j are,

$$y_{ij} = \mathcal{N}_{ij} \frac{Y_{ij}}{\sqrt{M_* L}} \frac{e^{\frac{-\alpha_j L}{2}} - e^{\frac{-\alpha_i L}{2}}}{(\alpha_i - \alpha_j)}, \tag{2.11}$$

where the normalization factor is given by $\mathcal{N}_{ij}^2 = 4\alpha_i\alpha_j/[(1 - e^{-\alpha_i L})(1 - e^{-\alpha_j L})]$. This equation is valid for both the up- and down-type Yukawa interactions, with the appropriate α_j for the right-handed field in each case. The basic idea is that the third family fermions are more weakly coupled to ϕ, resulting in a large overlap between the doublets and singlets, and thus strong coupling to the Higgs, whereas the second and first generations couple more strongly to ϕ, and thus have narrower profiles with exponentially suppressed overlaps and hence smaller interactions with the Higgs.

The model contains nine parameters (three α_{q_i}, three α_{u_i}, and three α_{d_i}) and to be considered successful, must fit the six quark masses and three real CKM angles with all of the widths of $\mathcal{O}(1)$. Generally, there is some tension in successfully generating the flavor observed in nature. The large top mass requires that the u_3 and q_3 zero modes be rather broad, which tends to generate large entries in the 13 and 23 entries of the mass matrices. Working only at the level of order of magnitudes, we find that for $M_* L = 10$, one can successfully realize quark flavor with widths ranging from 1/2 (for u_3) to 3 (for d_2 and q_1), with the Y_{ij} ranging from about 3 (for Y^u_{33}) to 1/3 (for $Y^{u(d)}_{23}$ and $Y^{u(d)}_{13}$). For $M_* L = 20$ one has widths from 1/5 to 3/2, with the same range of Y.

The lepton sector can be constructed by introducing the lepton doublets, l_i, right-handed charged singlets, e_i, and three right-handed neutrinos ν_i into the bulk, each coupled to the localizer. In the absence of any symmetries to protect them, we assume Majorana masses for the ν fields on the order of M_* which we now take to be 100 TeV. The 5d Yukawa interactions and bulk masses are,

$$\mathcal{L} = \frac{Y^e_{ij}}{\sqrt{M_*}}\, H\, \bar{l}_i\, e_j + \frac{Y^\nu_{ij}}{\sqrt{M_*}}\, H^c\, \bar{l}_i\, \nu_j + M^\nu_{Rij}\, \nu^c_i\, \nu_j + h.c.\,, \tag{2.12}$$

where $M^\nu_{Rij} \sim 100$ TeV are the Majorana masses for the right-handed neutrinos, and need not be diagonal in the same basis as the interactions with ϕ. The interactions of the zero modes include Yukawa interactions (y^e and y^ν) suppressed by the overlap of the zero-mode wave functions, as in eq. (2.11). When the Higgs acquires a VEV, this results in Dirac masses for the both the charged and neutral leptons. The charged lepton mass matrix can be diagonalized as was done for the quarks, but the neutrinos are more conveniently analyzed by first integrating out the heavy right-handed neutrinos. This results in effective Majorana masses for the left-handed neutrinos,

$$M^\nu_{Lij} = v^2 y^\nu_{ik}\, (M^\nu_R)^{-1}_{kl}\, y^{\nu*}_{lj}\,. \tag{2.13}$$

We attempt to understand lepton flavor by building a hierarchy into the 4d Yukawa interactions, y^e and y^ν, arising from the exponentially suppressed overlaps of the left- and right-handed lepton wave functions. We find it is generically easy to produce a heaviest neutrino relevant for atmospheric neutrino oscillation by simply arranging the wave functions such that $y^\nu_{23} \sim y^\nu_{33} \sim 10^{-5}$, which results in a neutrino with mass $m^2 \sim 10^{-3}\mathrm{eV}^2$ which is almost an equal mixture of ν_μ and ν_τ (some fine-tuning is required for the mixing to be near maximal). A small mixing angle solution for the solar neutrinos may then be produced by introducing much smaller entries $y^\nu_{11} \sim y^\nu_{12} \sim y^\nu_{22} \sim 10^{-6}$, producing a neutrino with mass $m^2 \sim 10^{-5}\mathrm{eV}^2$ which is almost entirely an electron neutrino, with small ν and τ components such that $\sin^2\theta \sim 10^{-2}$. The third neutrino is generally light and is largely the mixture of ν_τ and ν_μ orthogonal to the heaviest neutrino. This scheme can be realized within the context of eq. (2.11) when $M_* L = 10$ for widths varying between about 1 (for e_3) to about 4 (for ν_2 and l_1), and the underlying Y range between about 3 and 1/4. When $M_* L = 20$, we find that we need widths between about 1/2 (for e_3) to 5/2 (for ν_2 and l_1) with the same range of Y. The large mixing angle solution to the solar neutrino problem is difficult to realize in this scenario.

2.3 Localized Higgs VEV

Now we present what we believe is the most attractive scenario for extra-dimensional flavor: the possibility that the Higgs VEV is confined to one of the orbifold fixed points. This could be accomplished in a number of different ways. One option is that the Higgs field is simply confined to the boundary, and thus EWSB occurs only at a single point in the extra dimension. Another idea is that the Higgs is a bulk field, with a positive bulk mass^2 such that it does not develop a bulk VEV, but a separate negative mass^2 term exists on the boundary, and again the EWSB VEV develops only close to the boundary. A final scenario has the Higgs field in the bulk, coupled to more bulk fields (such as the localizer ϕ) which have VEV's which are functions of y and trigger EWSB only in a limited region of y. Generally one would expect some fine tuning associated with any of these options, since both bulk and boundary masses would naturally be of order $M_* \gg v$.

The fermions will be localized as before with $f_\psi \sim \mathcal{O}(1)$, which again will result in $\mathcal{O}(1)$ widths for their exponential zero mode profiles. If the Higgs and its VEV are, for example, confined to $y = 0$, the 5d mass terms for fermions are,

$$\mathcal{L} = \left\{ \frac{Y^d_{ij}}{M_*} \langle H \rangle \overline{q}_i d_j + \frac{Y^u_{ij}}{M_*} \langle H^c \rangle \overline{q}_i u_j \right\} \delta(y) + h.c. \tag{2.14}$$

(Note, the NDA estimate for the Y is on the order of $6\pi^2$ [14] but we will continue to assume $Y \sim 1$ as we are assuming a weakly coupled threshold at M_*). The effective 4d masses for the zero modes is equal to a product of the wave functions for those modes evaluated at $y = 0$,

$$\frac{m^d_{ij}}{v} = \frac{Y^d_{ij}}{M_*}\, \psi^0_{q_i}(0)\; \psi^0_{d_j}(0)\,, \qquad \frac{m^u_{ij}}{v} = \frac{Y^u_{ij}}{M_*}\, \psi^0_{q_i}(0)\; \psi^0_{u_j}(0)\,. \tag{2.15}$$

In an abbreviated notation in which we write the zero mode wave function at $y = 0$ as the field itself, $i = \psi_i(0)$, we thus have the following 4d mass matrices,

$$\frac{m^u}{v} \sim \begin{pmatrix} q_1 u_1 & q_1 u_2 & q_1 u_3 \\ q_2 u_1 & q_2 u_2 & q_2 u_3 \\ q_3 u_1 & q_3 u_2 & q_3 u_3 \end{pmatrix}, \qquad \frac{m^d}{v} \sim \begin{pmatrix} q_1 d_1 & q_1 d_2 & q_1 d_3 \\ q_2 d_1 & q_2 d_2 & q_2 d_3 \\ q_3 d_1 & q_3 d_2 & q_3 d_3 \end{pmatrix}, \tag{2.16}$$

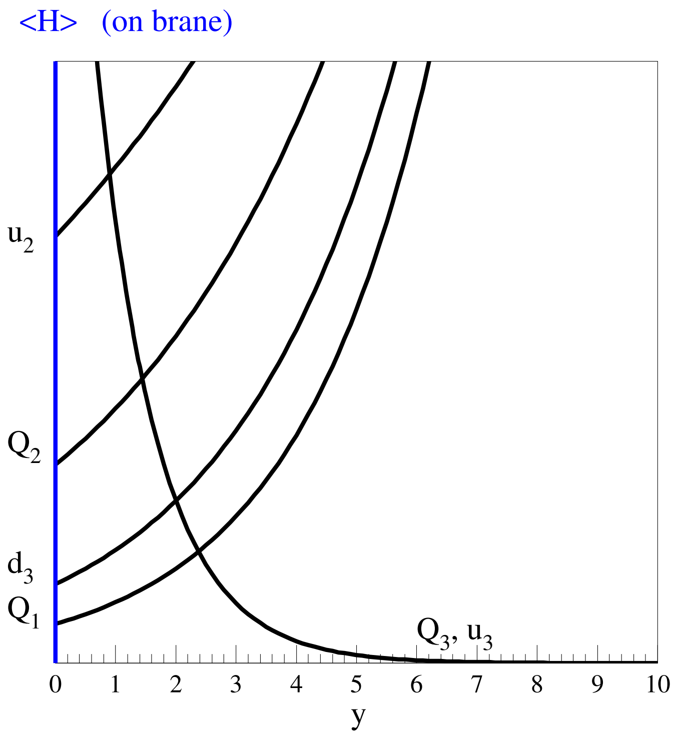

Figure 2: Zero mode profiles for some of the quarks, for the model with the Higgs VEV localized at $y = 0$.

where we have suppressed the underlying ($\mathcal{O}(1)$) 5d interactions, Y_{ij}, which multiply the corresponding entry in each matrix. The full matrices are thus generically of rank 3. Assuming there is a significant hierarchy as one moves along the rows and columns, this implies the simple relation between the three Cabibbo elements, $V_{ub} \sim V_{us}V_{cb}$. A further implication is that the contributions from the up and down sectors to the CKM matrix will be about equal in magnitude, in contrast to flavor symmetry models. The matrices are full in the sense that there are no negligible entries, so for general complex Y^u and Y^d, we should be able to realize CP violation to the extent required by measurements of ϵ_K [19].

We realize the large top Yukawa coupling by localizing q_3 and u_3 at the Higgs boundary by choosing $f_{q_3} \sim f_{u_3} \sim 1$, which results in $y^u_{33} \sim 1$. Note that we obtain the correct top mass without fine-tuning simply by requiring that one quark doublet and one (up-type) quark singlet are localized on the Higgs boundary. This is in contrast to the AS model where a q and u must be placed very close to one another. We localize the zero modes for all of the other fermions at $y = L/2$, and adjust the α_i in order to generate the observed Yukawas. For $M_*L = 10$, this results in widths ranging from about 1/3 (for u_2) to 1 (for u_1), and forces us to invoke 5d Yukawa couplings ranging from about 1/3 to 1. The resulting profiles for some of the zero modes are shown in figure 2.

Once again, we introduce three lepton doublets, charged singlets, and neutral singlets into the bulk, coupled to ϕ. We continue to assume right-handed neutrino masses on the order of 100 TeV. The 5d mass terms are,

$$\mathcal{L} = \left\{ \frac{Y^e_{ij}}{M_*} \langle H \rangle \, \bar{l}_i \, e_j + \frac{Y^\nu_{ij}}{M_*} \langle H^c \rangle \, \bar{l}_i \, \nu_j \right\} \delta(y) + M^\nu_{Rij} \, \nu^c_i \, \nu_j + h.c. \tag{2.17}$$

Moving to the Kaluza-Klein description, the zero modes for the left-handed neutrinos have Dirac masses with the entire tower of right-handed neutrino modes. The spacing in this tower will not be the compactification scale $1/L$ but characteristic of the width of the localized wave function. The contributions to the low energy neutrino masses will differ from those estimated below (where we only take into account zero modes) by coefficients of order unity, which is to the accuracy we are currently working.

The Dirac masses for the charged and neutral leptons are again proportional to wave functions evaluated at $y = 0$,

$$\frac{m^e_{ij}}{v} = \frac{Y^e_{ij}}{M_*} \, \psi^0_{l_i}(0) \; \psi^0_{e_j}(0) \, , \qquad \frac{m^\nu_{ij}}{v} = \frac{Y^\nu_{ij}}{M_*} \, \psi^0_{l_i}(0) \; \psi^0_{\nu_j}(0) \, , \tag{2.18}$$

which for the charged leptons may simply be diagonalized. Once again, we integrate out the heavy singlet neutrinos, resulting in an effective Majorana mass matrix for the left-handed neutrinos,

$$M^\nu_L = m^\nu \, \frac{1}{M^\nu_R} \, m^{\nu\dagger} \, . \tag{2.19}$$

We choose the couplings to the localizer such that e_3 is localized around the Higgs VEV, and all of the other leptons are localized around $y = L/2$. For $M_* L = 10$, we can realize the small mixing angle solution outlined in section 2.2 by choosing widths ranging from $1/2$ (for e_2) to 2 (for ν_2) and invoking 5d Yukawa interactions ranging from about $1/3$ to 3. Again, it proves somewhat difficult to realize the large mixing angle solution.

2.4 Constraints and the hierarchy solution

Theories in which fermions live at different locations in an extra dimensions are subject to constraints from flavor and CP violation arising from the higher KK modes of the gauge fields [20], whose interactions depend on the location and shape of the fermion wave function. After performing the CKM rotation into the quark mass basis, this results in flavor-changing neutral currents (FCNC's) at tree level. While these interactions are suppressed by the compactification scale M_c that sets the gauge boson KK masses, they may still be competitive with the SM predictions, which occur only through loops.

For simplicity we consider only the gluon KK modes, as they have the strongest couplings of the SM gauge fields, and flavor mixing in the first two generation down-

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type quarks,[3] which is expected to result in the strongest constraints. The wave functions for the $n > 0$ KK gluons are $\psi_A^n(y) \sim \cos[2\pi ny/L]$ with corresponding masses $M_n = 2\pi n/L = 2\pi n M_c$. The interaction between the nth KK gluon ($G_\mu^{(n)}$) and the strange and down quarks are,

$$\mathcal{L} = -\sqrt{2} g_S G_\mu^{(n)} \left(\overline{d}\,\overline{s}\right) \gamma^\mu T^a \left[D_L P_L + D_R P_R\right] \begin{pmatrix} d \\ s \end{pmatrix}, \tag{2.20}$$

where the matrices $D_{L(R)}$ are defined by

$$D_L = L_d \begin{pmatrix} c_1^{L(n)} & 0 \\ 0 & c_2^{L(n)} \end{pmatrix} L_d^\dagger, \qquad D_R = R_d \begin{pmatrix} c_1^{R(n)} & 0 \\ 0 & c_2^{R(n)} \end{pmatrix} R_d^\dagger, \tag{2.21}$$

a product of the left- (right-) handed down quark rotations L_d (R_d) from interaction to mass basis, and the couplings of the nth KK gluon to the left- (right-) handed quark zero modes,

$$c_i^{L(n)} = \int_0^{L/2} dy\ \cos\left[\frac{2\pi n}{L} y\right] \left|\psi^0(y)\right|^2 . \tag{2.22}$$

In [20] the left-left current contribution of the $\Delta S = 2$ portion of these interactions to Δm_K and $|\epsilon_K|$ was considered. The requirement that the extra dimensional contribution is no larger than the experimentally determined values [22] yields the constraints,[4]

$$M_c \gtrsim 160 \text{ TeV} \sqrt{\sum_{n=1}^{n^*} \frac{\text{Re}\left[D_{L\{12\}}^2 + D_{R\{12\}}^2 + 14.8 D_{L\{12\}} D_{R\{12\}}\right]}{n^2}}, \tag{2.23}$$

$$M_c \gtrsim 2800 \text{ TeV} \sqrt{\sum_{n=1}^{n^*} \frac{\text{Im}\left[D_{L\{12\}}^2 + D_{R\{12\}}^2 + 14.8 D_{L\{12\}} D_{R\{12\}}\right]}{n^2}}, \tag{2.24}$$

where we have used the vacuum insertion approximation and the factor of 14.8 accounts for the difference in the hadronic matrix element for a left-right as opposed to a left-left (or right-right) current operator [23] as well as a relative factor of two in the effective hamiltonian. The sum over KK modes is explicitly cut-off at $n^* \sim M_* L/(2\pi)$ to avoid counting the modes with mass greater than M_*. In fact for all of the models we will consider there is very little sensitivity to n_* because of the $1/n^2$ suppression in the sum, as well as an additional suppression because the high frequency modes tend to average to zero over the quark wave functions.

[3] Similar FCNC bounds on extra-dimensional lepton flavor models were considered in [21].

[4] Note that our definition of M_c differs by a factor of 2π from that of [20]. Our results for the left-left current constraints are consistent with [20] to within about 10%, well within the theoretical uncertainties.

Quantity	AS Model	Higgs in the Bulk	Higgs on a Brane
M_c from Δm_K	120 TeV	5 TeV	13 TeV
M_c from ϵ_K	2100 TeV	80 TeV	230 TeV
M_*	$50 \times M_c$	$10 - 20 \times M_c$	$10 - 20 \times M_c$

Table 1: Limits from Kaon measurements on the three models described in the text.

Armed with the model-independent constraints eqs. (2.23) and (2.24), we can now derive constraints on specific models of large extra dimensions. One can derive analytic expressions for the c^n constants for all of the models we have considered, but as the expressions are somewhat unwieldy and not very illuminating, instead we prefer to quote the resulting bounds. The limits on M_c are presented in table 1 for the three models described above. For reference, we also show the expected relationship between the compactification and fundamental (assumed to be related to the wave function width) scales. We note that our bounds on the AS model are a factor of about 50 more stringent than those derived in [20] which used a different definition of the AS model, and included only the left-left flavor violating currents. In considering the bounds from $|\epsilon_K|$, one should keep in mind that the the AS solution contains approximate zeros in the down quark mass matrix which would allow one to approximately rotate all of the CP-violating phases away. This feature could allow us to interpret the bounds from $|\epsilon_K|$, as a *prediction* of that model for M_c, as this extra-dimensional contribution may be able to explain the experimental measurements, though of course this would be a coincidence.

As the table shows, the models we have constructed in which the quarks and leptons live on one or the other of the orbifold fixed points have significantly weaker bounds on M_c than the AS model. This can be understood largely from the fact that in the orbifold models the first and second generation down-type quarks are localized about the same point, with differences in masses and mixings arising from the different widths of the wave functions, whereas in AS the quarks are localized at different points and thus for the lower KK modes of the gluon (which dominate the sum in eqs. (2.23) and (2.24) the couplings to the two quarks are more equal, and thus the flavor-violation less pronounced. Furthermore, the AS model requires the fundamental scale be considerably higher than the other models, because it must actually space the multiple fermions away from each other to get small masses *and* mixing angles.

If one makes the extra dimension in AS a bit larger, one can incorporate their solution to the problem of proton decay via higher dimension operators. Their solution, separating quarks and leptons in the bulk, is not easily adapted into our framework and thus we require further ingredients (for example, imposing additional gauge symmetries could forbid the dangerous operators) to be consistent with proton decay constraints.

3. Supersymmetric models and y-dependent flat directions

We now promote the above models to supersymmetrized versions using the superfield notation of [10] for dimensions greater than four (which we review below). We find that using odd mass terms is enough to generate the complete Yukawa hierarchy. We also find scalar VEV profiles which preserve $\mathcal{N}=1$ supersymmetry and can be used to localize chiral superfields. While the restrictive nature of 5d supersymmetry makes it difficult to construct realistic models, we do outline a working model in this context compactified on an orbifold. Finally, we present a model compactified on S^1 where chirality is simply introduced by introducing a brane containing chiral matter. Vector-like fields in the bulk mix with the chiral fields on the brane and a scalar VEV "projects" the massless chiral fermions into the bulk.

3.1 Superspace for five-dimensional supersymmetry

Using the notation of [10], we formulate a 5d supersymmetric gauge theory in the language of $\mathcal{N}=1$ 4d superfields. This allows us to use the powerful superfield machinery to analyze the conditions under which one supersymmetry is preserved in the 4d effective theory. We find y-dependent flat directions which then can be used as tools for model-building.

3.1.1 Hypermultiplets

Chiral superfields in five dimensions come in pairs, called hypermultiplets. A free hypermultiplet was first described in the following notation in [24]:

$$\int d^4\theta \left(H^\dagger H + H^{c\dagger} H^c\right) + \int d^2\theta \; H^c \left(\partial_y + m\right) H + \text{h.c.} \tag{3.1}$$

If the fifth dimension is compactified on a $\mathcal{Z}_2$ orbifold, then H and H^c must transform oppositely under the discrete symmetry and thus $m = 0$. Another option is to give the hypermultiplet a mass which is odd under the $\mathcal{Z}_2$ with one chiral superfield odd and the other even. This mass term preserves the full 5d supersymmetry everywhere except at the boundaries, where it preserves half.

3.1.2 An abelian gauge multiplet

The 5d gauge sector consists of a vector superfield V whose components are the four-dimensional part of the vector gauge field A^μ, the left-handed gaugino λ_L, and an auxiliary field D, and a chiral superfield Φ whose components are a complex scalar $\phi = (\Sigma + iA_5)/\sqrt{2}$ (containing both the fifth component of the vector field A_5 and the real scalar Σ), the right-handed gaugino λ_R, and a complex auxiliary field F. The 5d lagrangian density is given by

$$\int d^4\theta \frac{1}{g^2}\left(\Phi^\dagger\Phi - \sqrt{2}\left(\Phi^\dagger + \Phi\right)\partial_y V - V\partial_y^2 V\right) + \int d^2\theta \frac{1}{4g^2} W_\alpha W^\alpha + \text{h.c.} \tag{3.2}$$

While this lagrangian is only manifestly 4d Poincaré invariant, it is in fact invariant under the full 5d Poincaré symmetry. It is also invariant under the 5d gauge transformations: $V \to V + \Lambda^\dagger + \Lambda$ and $\Phi \to \Phi + \sqrt{2}\partial_y \Lambda$, as well as the full $\mathcal{N} = 2$ supersymmetry transformations [10].

3.1.3 Charged matter

A hypermultiplet of charge Q consists of two chiral superfields H and $\tilde{H}$ with scalar components h and $\tilde{h}$, fermionic components ψ_h and $\psi_{\tilde{h}}$, and auxiliary fields F_H and $F_{\tilde{H}}$ and the following terms in the lagrangian:

$$\int d^4\theta \left[H^\dagger e^{-QV} H + \tilde{H}^\dagger e^{QV} \tilde{H}\right] + \int d^2\theta \left[\tilde{H}\left(\partial_y + m - \frac{Q}{\sqrt{2}}\Phi\right) H\right] + \text{h.c.} \tag{3.3}$$

Generalizing to more than one hypermultiplet is trivial. For hypermultiplets of the same charge, m can be a matrix with non-trivial flavor structure. Under gauge transformations, the hypermultiplet transforms as $H \to e^{Q\Lambda} H$, and $\tilde{H} \to e^{-Q\Lambda} \tilde{H}$.

3.1.4 Coupling to branes/boundaries

One of the reasons this notation is so powerful is that it makes coupling bulk fields to branes trivial. For example, a superpotential coupling of a component H of an uncharged hypermultiplet to a brane at $y = 0$ would look like

$$\int d^2\theta\, JH\delta(y)\,, \tag{3.4}$$

where J is a gauge invariant operator made up of brane fields and/or numerical constants. A Fayet-Iliopoulos term on a brane at $y = 0$ looks like

$$\int d^4\theta\, 2\,\xi\, V\, \delta(y)\,, \tag{3.5}$$

while adding charged fields $X, \tilde{X}$ (with charges ± 1) to a brane at a point $y = L/2$ requires

$$\int d^4\theta\, \left(X^\dagger e^{-V} X + \tilde{X}^\dagger e^{V} \tilde{X}\right) \delta(y - L/2)\,. \tag{3.6}$$

When translated into component language, this notation reproduces the results of [25].

3.1.5 Flat directions

We now have the machinery needed to look for y-dependent flat directions which preserve the 4d $\mathcal{N} = 1$ supersymmetry. We simply need to solve the F- and D-flat conditions. Before we do, we remind the reader that our fifth dimension is compact. We are interested in both compactification on a simple circle (S^1), with $-L/2 < y \le L/2$, and on an orbifold ($S^1/\mathcal{Z}_2$), with the same range for y but with

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y and $-y$ identified. In the latter case, the superfields H and V are even under the $\mathcal{Z}_2$ and $\tilde{H}$ and Φ are odd. This has the consequence (as in the previous section) that a normal mass term connecting H with $\tilde{H}$ is forbidden while an odd mass term (proportional to $\epsilon(y)$) is allowed.

First let us look at the case with a U(1) vector multiplet with a Fayet-Iliopoulos term and matter with charges $\pm Q$ on branes (or orbifold fixed points) at $y=0$ and $y=L/2$ respectively. The D-flat condition requires

$$-D = \left[2\xi\delta(y) + \frac{g\,Q}{2}\left(|\tilde{X}|^2 - |X|^2\right)\delta(y-L/2) + \partial_y\Sigma\right] = 0\,, \tag{3.7}$$

which is satisfied by the conditions $|\tilde{X}|^2 - |X|^2 = -4\xi/gQ$ and $\Sigma = \Sigma_0 + \xi\epsilon(y)$. In the case of the orbifold, $\Sigma_0 = 0$ since Σ is odd around the points $y = 0, L/2$. It is simply a degree of freedom which is projected out of the theory by the orbifold [10].

As can be seen from eq. (3.3), Φ can play the role of the localizer field[5] with standard model matter (and their mirror partners) as hypermultiplets in the bulk. The lagrangian contains $\psi_{\tilde{q}}(D_y + Q\Sigma(y)/2)\psi_q$ which localizes the zero mode of the quark ψ_q(mirror-quark $\psi_{\tilde{q}}$) where $\Sigma(y)$ crosses zero with a positive (negative) slope. In a compact space, zero modes only exist for $\Sigma_0 = 0$. In the orbifold case this condition, as well as the removal of the $\tilde{q}$ zero mode, is guaranteed by the boundary conditions. In the S^1 case this could be guaranteed by a soft mass for Σ on either brane. If Σ_0 is non-zero, the lightest mode mass goes as $\sqrt{\xi^2 - \Sigma_0^2}\,e^{-(\xi-\Sigma_0)L/2}$ for Σ_0 at least somewhat smaller than ξ.

More interesting VEV profiles can appear if we include a hypermultiplet in the flat directions. Using equations (3.2) and (3.3), we look for solutions to the differential equations resulting from imposing the F- and D-flatness conditions $D^2 = |F_H|^2 = |F_{\tilde{H}}|^2 = |F_\Phi|^2 = 0$, where,

$$\begin{aligned}
-F_\Phi^* &= -\frac{g\,Q}{\sqrt{2}}\tilde{h}h \\
-F_{\tilde{H}}^* &= \left[\partial_y + m - \frac{Qg}{\sqrt{2}}\phi\right] h \\
-F_H^* &= \left[-\partial_y + m - \frac{Qg}{\sqrt{2}}\phi\right]\tilde{h} \\
-D &= \frac{g\,Q}{2}\left(|\tilde{h}|^2 - |h|^2\right) + \partial_y\Sigma\,.
\end{aligned} \tag{3.8}$$

As it turns out for the S^1 and $S^1/\mathcal{Z}_2$ geometries, the only solutions are $\Sigma = \Sigma_0$ and $\phi = 0$ respectively with all other scalar fields zero. This is because the compactification of the extra dimension requires solutions which are periodic, and while

[5] Localizing chiral superfields requires a straight-forward generalization of the procedure for localizing a fermion zero mode. For details, see [6].

such solutions to eqs. (3.8) exist, they have VEV's which are singular at points in the extra dimension, and thus our effective theory description of the physics may not be applicable. In order to have nontrivial profiles valid within the context of the effective theory, we introduce a 3-brane located at $y = 0$ with a Fayet-Iliopoulos term (3.5), which modifies the D term equation as in (3.7). This in turn induces a discontinuity in the VEV of ϕ at the brane. The F_Φ and F_H equations may be satisfied by requiring $\tilde{h} = 0$, and the remaining two equations have solutions,

$$\begin{aligned} h(y) &= \frac{2\alpha}{g\cos[\alpha(y + L/2 - L\Theta(y))]} \\ \Sigma(y) &= \frac{\alpha}{g}\tan[\alpha(y + L/2 - L\Theta(y))] + \frac{m}{g}\,, \end{aligned} \tag{3.9}$$

where we have taken the charge $Q = 1$ and explicitly chosen a (5d) gauge to make $h(y)$ real and A_5 vanish. The parameter α is related to the magnitude of the Fayet-Iliopoulos term by,

$$\alpha = \sqrt{\frac{g\,\xi}{2\,L}}\,, \tag{3.10}$$

and nonsingular VEV's in the interval $-L/2 \leq y \leq L/2$ require $\alpha < \pi/L$.

Again, the profile for Σ acts as a "mass function" and will tend to localize the right-handed components of (positively charged) hypermultiplets about the point where the brane sits ($y = 0$) and the left-handed components at a point in the bulk where $\Sigma(y)$ crosses zero [6], with the KK tower masses given as eigenvalues of the supersymmetric QM hamiltonians, $-\partial_y^2 \mp g/\sqrt{2}(\partial_y\Sigma(y)) + g^2/2\Sigma^2(y)$. As an example in order to divine some general features, we consider the case where $m = 0$ (which would be enforced, for example, by orbifold boundary conditions) and will allow for the lightest modes in the KK decomposition to have zero mass. Our analysis is further simplified when α is small, which allows us to expand the profile for Σ as,

$$\Sigma(y) = \frac{\alpha^2}{g}\left[y + L/2 - L\Theta(y)\right]. \tag{3.11}$$

The zero mass solutions for a hypermultiplet (containing chiral multiplets $\tilde{\Psi}$ and Ψ) of charge Q are,

$$\psi^0_\pm(y) = \mathcal{N}_\pm \mathrm{Exp}\left[\pm Q\alpha^2\left(\frac{1}{2}y^2 + \frac{L}{2}y - Ly\Theta(y)\right)\right], \tag{3.12}$$

where $\mathcal{N}_\pm$ are chosen to correctly normalize the kinetic terms. In the limit of large L, these solutions look increasingly like an exponential centered at $y = 0$ and a gaussian centered at $y = L/2$, which is understandable because in that limit the corresponding hamiltonians look like a δ-function potential at $y = 0$ and a simple harmonic oscillator at $y = L/2$, each surrounded by large "potential barriers" that discourage the wave functions of the low mass modes from spreading.

If we now allow non-zero $\Delta m = m_H - m$, the situation changes in two important ways. The zero-crossing of the linear term in Σ will shift, which will translate the center of the part of the hamiltonian which looked like a harmonic oscillator (if Δm is large enough, the zero crossing may in fact disappear altogether). More importantly, the two fields which formerly corresponded to the right- and left-handed zero modes will now marry one another with some non-zero mass of $\mathcal{O}(\Delta m)$. However, provided L is large (and thus the potential barrier between the two localizing potentials in the corresponding Schrödinger problem is also large), the profiles for this pair of light modes remain localized as they were for the $m = 0$ case. Thus, the lightest modes of the KK spectrum are a pair of chiral superfields ($\tilde{H}^0$ and H^0) localized at $y = 0$ (with approximately exponential profile) and $y = L/2 - \Delta m\, Q/\alpha^2$ (with approximately gaussian profile), respectively.

3.2 Flavor from an odd mass term

The simplest and perhaps most attractive model of the sort we are discussing requires masses for hypermultiplets which are odd under the $\mathcal{Z}_2$ of the orbifold.[6] The hypermultiplets are the SM quarks and leptons and their 5d chiral partners. The odd mass localizes the chiral zero modes of the matter fields at one of the orbifold fixed points, depending on the sign of the mass-term step function.

The model reproduces (a supersymmetrized version of) the model in section 2.3 with the scalar profile idealized to a step function. The Higgses are now chiral superfields and can be localized in the same way as the matter fields. The Yukawa interactions are forbidden by $\mathcal{N} = 2$ supersymmetry, but may be explicitly introduced on the branes. Thus the most successful model of the previous section can be put into a supersymmetric context and thereby decouple the gauge hierarchy problem from the generation of Yukawa suppression. In fact, because generic supersymmetric theories have two Higgs doublets, one to generate up-type quark (and, if relevant, Dirac neutrino) masses, and the other to generate down-type quark and charged lepton masses, we have an additional freedom in constructing flavor in a supersymmetric context in the choice of ratio of the Higgs VEV's, $\tan\beta = v_u/v_d$. This allows one, for example, to partially or completely generate the hierarchy between the top and bottom masses by the choice of $\tan\beta$, and allows some more flexibility in generating realistic flavor.

The next question is whether or not one can promote the above mass term to a field such that one can produce successful models of flavor with localized fermions zero modes by the y-dependent profiles described above. The short answer is no. Coupling a hypermultiplet to the Σ field in the superpotential requires the hypermultiplet to be charged under a gauge symmetry. Taking that gauge symmetry to be U(1), the couplings of matter fields to Σ, and thus the width of their wave functions, are proportional to the charge of the hypermultiplet in question.

[6]We thank Andrea Romanino, who was the first to point out this possibility to us.

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If one wants different widths for different generations, the fields must have different charges. However, this forbids most or all of the Yukawa couplings in the 5d theory. One could choose charges Q such that $Q_{q_i}+Q_{u_i}+Q_{h_u}=0$ and $Q_{q_i}+Q_{d_i}+Q_{h_d}=0$, where $i=1,2,3$ is the generation index, and so at best one can get the right mass hierarchies in both the up and down sectors (with the μ term forbidden by the U(1) symmetry). However, the Yukawa matrices will already be diagonal and thus the CKM matrix is the identity matrix. One can remedy this situation by noticing that the boundary fields required to produce the Σ profile break the U(1) gauge symmetry spontaneously. This field can be used to produce non-renormalizable operators which could allow mixing terms once the field's VEV is inserted. The result is a hybrid extra-dimensional/Froggatt-Nielsen mechanism for fermion masses. While this idea seems workable, the resulting models are more in the Froggatt-Nielsen spirit than an extra-dimensional one, so we will not pursue them here.

3.3 Compactifying on S^1

An alternative to the orbifold is to work with the extra dimension compactified on S^1, and introduce chiral matter explicitly on a 3-brane. To illustrate how this works, let us consider a bulk hypermultiplet containing chiral multiplets Ψ and Ψ^c. We include a 3-brane at $y=0$ on which lives a chiral superfield η, and include a brane-coupling between η and Ψ^c,

$$\int dy \int d^2\theta \, \Psi^c(\partial_y+m)\Psi + M\,\eta\,\Psi^c\,\delta(y)\,. \tag{3.13}$$

Without the orbifold, Ψ is allowed an ordinary mass m. Ignoring the brane coupling for the moment, we consider the case in which Ψ is charged with charge Q under a bulk U(1) whose Σ is given the profile of eq. (3.9). Its lightest KK mode Ψ^0 will tend to localize around the zero crossing of the function $(Q\times\langle\Sigma\rangle(y)-m)$. Of course, it will have some non-zero mass m_0 with $\Psi^{c\,0}$, which will tend to localize around the brane (the locations can be reversed by adjusting the sign of Q and/or m). If we now turn on the brane coupling, the net result will be one massless field and one with mass $\sqrt{M_0^2+m_0^2}$ (with M_0 given by the overlap of the $\Psi^{c\,0}$ wave function with the brane), each of which is a mixture of the bulk light mode Ψ_0 and the brane field η. The composition of the zero mass field will be

$$-\frac{M_0}{\sqrt{M_0^2+m_0^2}}\Psi_0+\frac{m_0}{\sqrt{M_0^2+m_0^2}}\eta\,, \tag{3.14}$$

indicating that provided $M_0\gg m_0$, we have essentially recovered a chiral field in the bulk (though with some small component living on the brane). The chiral fields on the brane have been "projected" from the brane into the bulk by appropriate mixing with bulk fields.

We can use this tool to avoid the problems of the $S^1/\mathcal{Z}_2$ models in the previous section. We introduce a brane containing the entire MSSM chiral superfield sector, with brane couplings to an entire MSSM hypermultiplet sector in the bulk.[7]

In order to allow for Higgs couplings, we assign each generation the same charges for a given type of field, for example: $Q_q = +1/2$, $Q_{u^c} = +1$, $Q_{d^c} = -2$, $Q_l = +1/2$, $Q_{e^c} = -2$, $Q_{n^c} = +1$, $Q_{H_u} = -3/2$ and $Q_{H_d} = +3/2$. These charges allow inter-generational couplings to the Higgses (on the branes), and results in the bulk light modes for the q, u^c, l, n^c, and H_d fields living at various points in the bulk (with positions determined by the corresponding hypermultiplet masses) and the e^c, d^c, and H_u fields all living on the brane. The right-handed neutrino masses are now forbidden by the U(1) symmetry, but could be generated by a non-renormalizable superpotential term such as HHn^cn^c. By appropriately choosing the bulk masses, we may adjust the overlaps of the left-handed fields with the right-handed fields and Higgses, and thus realize viable flavor. This mechanism has something in common with both the AS mechanism in that one sees suppression from right- and left-handed fields overlapping, and also some features of suppression due to the overlap with the Higgs present in the models of section 2.3 and [6].

4. Supersymmetry breaking

Having successfully constructed supersymmetric theories in which flavor is generated by an extra-dimensional mechanism, it is important to also consider how supersymmetry is broken. A generic supersymmetry-breaking mechanism could lead to off-diagonal entries in the sfermion mass matrices. The simplest way to avoid this supersymmetric flavor problem is to break supersymmetry in such as way as to guarantee that all sfermions of the same charge have approximately degenerate masses. This insures that after the rotation from gauge to mass eigenstates required to diagonalize the fermion masses, the sfermion masses remain diagonal. Since we have already introduced an extra dimension, we briefly consider two extra-dimensional supersymmetry-breaking mechanisms: Scherk-Schwarz breaking [26] by twisted boundary conditions (as realized in [27]), and gaugino-mediation [28, 29]. A third possibility resulting from a non-supersymmetric vacuum configuration of domain walls was explored (including flavor issues) in [31].

4.1 Scherk-Schwarz breaking

Any of the flavor models of the previous section can incorporate supersymmetry breaking by modifying the orbifold boundary conditions on the components of the

[7] For producing the right masses in the charged sector, it is in fact not necessary to put an entire MSSM hypermultiplet sector in the bulk. Simply a set of (what makes up) 10's and their conjugates will do.

superfields such that masses of the superpartner zero-modes are lifted to weak scale values. The model, and discussion, follows closely the one proposed in [27]. As before, we break $\mathcal{N}=2$ down to $\mathcal{N}=1$ by requiring that under the identification $y \leftrightarrow -y$ the superfields transform as,

$$\begin{pmatrix} V \\ \Phi \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} V \\ -\Phi \end{pmatrix} (x^\mu, y)\,,$$
$$\begin{pmatrix} \Psi \\ \Psi^c \end{pmatrix} (x^\mu, -y) = \begin{pmatrix} \Psi \\ -\Psi^c \end{pmatrix} (x^\mu, y)\,, \tag{4.1}$$

where Ψ and Ψ^c together form one of the matter hypermultiplets. Under $y \leftrightarrow y+2\pi R$, the two gauginos and two sfermions are twisted into each other by an element of the $\mathrm{SU}(2)_R$ symmetry of the 5d theory,

$$\begin{pmatrix} \lambda_L \\ \lambda_R \end{pmatrix} (x^\mu, y+2\pi R) = e^{-i2\pi\alpha\sigma_2} \begin{pmatrix} \lambda_L \\ \lambda_R \end{pmatrix} (x^\mu, y)\,,$$
$$\begin{pmatrix} \widetilde{f} \\ \widetilde{f}^c \end{pmatrix} (x^\mu, y+2\pi R) = e^{-i2\pi\alpha\sigma_2} \begin{pmatrix} \widetilde{f} \\ \widetilde{f}^c \end{pmatrix} (x^\mu, y)\,, \tag{4.2}$$

where σ_2 is the Pauli matrix, and α is a dimensionless parameter specifying the amount of twisting. The vectors, gauge scalars, and fermions are untwisted.

The additional boundary conditions on the fields modify the KK expansion for the gaugino modes to,

$$\begin{pmatrix} \lambda_L \\ \lambda_R \end{pmatrix} (x^\mu, y) = \sum_n e^{-i\alpha y/R\sigma_2} \begin{pmatrix} \lambda_L^n \cos[ny/R] \\ \lambda_R^n \sin[ny/R] \end{pmatrix}, \tag{4.3}$$

which, substituted into the 5d action 3.2 and integrating over y results in universal masses α/R for the gaugino zero modes. Assuming a compactification scale close to the GUT scale, this requires $\alpha \sim 10^{-13}$ in order to have gaugino masses at the weak scale.

The scalar masses are slightly more subtle. First, we note that the matter fermions have untwisted boundary conditions, and so are localized exactly as before, with wave functions $F^n(y)$ for the fermions (including a zero mode) and wave functions $G^n(y)$ for the mirror fermions. In this basis the KK expansion for the sfermions is

$$\begin{pmatrix} \widetilde{f} \\ \widetilde{f}^c \end{pmatrix} (x^\mu, y) = \sum_n e^{-i\alpha y/R\sigma_2} \begin{pmatrix} \widetilde{f}_n \, F^n(y) \\ \widetilde{f}_n^c \, G^n(y) \end{pmatrix}. \tag{4.4}$$

Inserting this expansion into the 5d the kinetic term produces universal sfermion masses α^2/R^2. Flavor-dependent corrections to the wave functions and masses will appear at order α/R and thus are negligibly small.

This model manages to generate the correct fermion spectrum while avoiding supersymmetric flavor problems. This is in contrast to a Froggatt-Nielsen type of

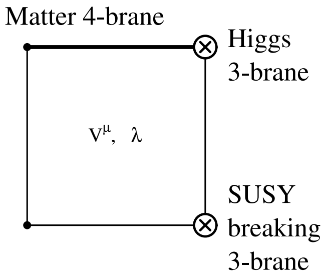

Figure 3: Schematic of the 6d model, indicating the locations of the Higgs and supersymmetry-breaking 3-branes, the matter 4-brane, and the gauge and gaugino fields in the bulk.

mechanism which, if the flavor-breaking scale is at least somewhat below the compactification scale, will produce flavor-violation in the scalar sector through renormalization group running. Unfortunately, the mechanism of flavor generation has virtually no impact on the superpartner spectrum and thus would be difficult to study experimentally at energies far below the compactification scale.

4.2 Gaugino mediation

In order to simply imbed the model of section 3.2 in a model of gaugino-mediation, we consider a theory with *two* extra dimensions compactified as $T^2/\mathcal{Z}_2$, a 2-torus with two points mapped into each other by a π rotation in the plane of the compact dimensions identified. The coordinates in the extra dimensions can be expressed as a 2-vector $\vec{y} = (y_1, y_2)$, with the physical space lying inside the rectangle bounded by the four orbifold fixed points at $(0,0)$, $(\pi R_5, 0)$, $(0, \pi R_6)$, and $(\pi R_5, \pi R_6)$ [30]. For simplicity, we consider the case where the two compact dimensions are orthogonal, and both radii are equal, $L = \pi R_5 = \pi R_6$. The gauge fields live in the entire 6d bulk, with the quarks and leptons confined to a 4-brane stretching between two of the orbifold fixed points (with zero modes localized along the small brane direction in order to produce flavor as in section 3.2), and the Higgses live in a 3-brane located at one of these two points. Supersymmetry is broken at one of the two-fixed points outside of the matter-brane. The situation is shown schematically in Figure 3.

If we parameterize the supersymmetry-breaking by a chiral superfield X whose auxiliary component F_X has a non-vanishing VEV, gauginos acquire a mass at tree-level from effective interactions such as,

$$\int d^6x \int d^2\theta \, \frac{\lambda_X}{M_*^3} X \, W^\alpha \, W_\alpha \, \delta(\vec{y}) \,, \tag{4.5}$$

where λ_X is a dimensionless coupling of order unity, and $\vec{y} = (y_1, y_2)$ are the coordinates in the extra dimensions. This results in a mass for the zero-mode gaugino of $\lambda_X \langle F_X \rangle / M_*^3 L^2$, suppressed by the volume of the extra dimension.

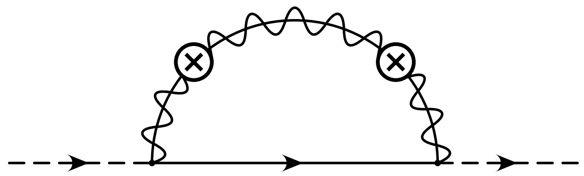

Figure 4: Loop diagram showing how gauginos carry supersymmetry-breaking information to the sfermions. The $\otimes$'s represent insertions of the operator in eq. (4.5).

The sfermions are prevented from getting masses (or A terms) directly from X by 6d locality, and must instead acquire masses at one loop from the gauginos through Feynman graphs such as that shown in figure 4. Below the compactification scale, the only relevant contribution from this graph has the gaugino zero-mode in the loop (which in fact corresponds to the usual renormalization group evolution of the sfermion mass induced by the gauginos in 4d). Since the zero-mode has flavor-blind couplings to the sfermions, this results in universal sfermion masses, as desired. However, above the compactification scale the higher KK modes of the gaugino will also contribute to the sfermion masses, and since they have wave functions which vary across the extra dimension, they couple flavor-diagonally, but not flavor-independently. This is potentially a problem, because after the rotation from the gauge to mass basis, the sfermions will, in general, pick up off-diagonal entries proportional to the mass^2 differences multiplied by rotation angles.

These contributions may be estimated by expressing the 6d gaugino propagator $\P[q; \vec{a}, \vec{b}]$ in mixed position and momentum space (see, for example [6]), and evaluating the 6d effective action at one loop, summing over all of the gaugino KK modes in the loop[8] and identifying the term relevant for sfermion masses,

$$\Gamma_6\left[\widetilde{f}^*, \widetilde{f}\right] = \int d^4x\, dy_1\, dy_2\, \widetilde{f}^*(x^\mu, y_1)\, \widetilde{f}(x^\mu, y_2)\, M^2(y_1, y_2)\,, \tag{4.6}$$

where $y_{1(2)}$ are positions along the matter 4-brane, and we have explicitly used the fact that a 4d mass term must be evaluated for both fields at the same 4d space-time point, and that the (s)fermions are confined to a 5d subspace. The coefficient $M^2(y_1, y_2)$ is,

$$M^2(y_1, y_2) = \frac{\alpha}{4\pi} M_{1/2}^2 \int d^4q \mathrm{Tr}\left[\frac{\not{q}}{q^2} \P[q; (L, y_1), \vec{0}]\P[q; \vec{0}, \vec{0}]\P[q; \vec{0}, (L, y_2)]\right], \tag{4.7}$$

where α is defined in terms of the 4d gauge coupling, and we have dropped Casimirs and factors of 2. One may then compactify down to four dimensions, inserting the wave functions for the (s)fermions and determining the effective mass of the sfermion

[8] In six dimensions, this introduces some dependence on how the sum over KK modes is cut-off; we have adopted a hard cut-off at M_*.

"zero-mode" at the compactification scale,

$$\widetilde{m}^2 = \int dy_1\, dy_2\, \psi^{0*}_{\tilde{f}}(y_1)\, \psi^{0}_{\tilde{f}}(y_2)\, M^2(y_1, y_2)\,. \tag{4.8}$$

The weak scale superpartner masses are then obtained by rotating to the quark mass basis, and applying the usual 4d renormalization evolution from the compactification scale to the weak scale.

For simplicity we assume $M_c \sim M_{GUT}$, with the gaugino masses given by a single parameter $M_{1/2}$. The boundary conditions for the scalar mass^2 matrices at M_c for the specific localized Higgs model described in section 3.2 results in,

$$\begin{array}{lll} \widetilde{m}^2_{q_1} \sim 0.024\, M^2_{1/2}\,, & \widetilde{m}^2_{u_1} \sim 0.023\, M^2_{1/2}\,, & \widetilde{m}^2_{d_1} \sim 0.023\, M^2_{1/2}\,, \\ \widetilde{m}^2_{q_2} \sim 0.040\, M^2_{1/2}\,, & \widetilde{m}^2_{u_2} \sim 0.072\, M^2_{1/2}\,, & \widetilde{m}^2_{d_2} \sim 0.025\, M^2_{1/2}\,, \\ \widetilde{m}^2_{q_3} \sim 0.140\, M^2_{1/2}\,, & \widetilde{m}^2_{u_3} \sim 0.140\, M^2_{1/2}\,, & \widetilde{m}^2_{d_3} \sim 0.030\, M^2_{1/2}\,, \end{array} \tag{4.9}$$

which after applying the CKM rotations and evolving down to the weak scale will result in, i.e.,

$$\delta^{LL}_{sd} \sim \frac{\widetilde{m}^2_s - \widetilde{m}^2_d}{\widetilde{m}^2_s} \times V_{us} \sim 5 \times 10^{-4}\,, \tag{4.10}$$

(with similar results for δ^{RR}), acceptably small [32]. And for the leptons we have,

$$\begin{array}{ll} \widetilde{m}^2_{l_1} \sim 0.016\, M^2_{1/2}\,, & \widetilde{m}^2_{e_1} \sim 0.017\, M^2_{1/2}\,, \\ \widetilde{m}^2_{l_2} \sim 0.026\, M^2_{1/2}\,, & \widetilde{m}^2_{e_2} \sim 0.050\, M^2_{1/2}\,, \\ \widetilde{m}^2_{l_3} \sim 0.026\, M^2_{1/2}\,, & \widetilde{m}^2_{e_3} \sim 0.140\, M^2_{1/2}\,. \end{array} \tag{4.11}$$

The much larger corrections to the masses of q_3, u^c_3, and e^c_3 are a direct result of those fields being localized around the Higgs brane, and thus having wave functions concentrated closer to the supersymmetry breaking brane. The Higgses receive negligibly small soft masses at M_c. Thus, we see that the flavor model has left an imprint of sorts on the sparticle mass spectrum.

The resulting weak scale sparticle masses (with the μ term fixed by the requirement of proper EWSB - for specific gaugino-mediation solutions to the μ-problem, see [6, 29, 33]) show some distinct differences from standard gaugino-mediation. First, the lightest superpartner is typically a neutralino as opposed to a stau, because of the additional contribution to stau masses in eq. (4.11). This feature allows us to more simply connect with a standard picture of cosmology. Further, there is non-degeneracy of the squarks and sleptons of different families, and for different chiralities, with the most pronounced difference for the third generation. This is a direct consequence of the fact that the extra dimensions play a nontrivial role both in generating flavor breaking and in supersymmetry breaking, and represents a way in which future experiments could make progress to unravel the flavor puzzle, by making precision measurements of the supersymmetry-breaking parameters.

5. Conclusion

In this article we have examined a variety of tools, both in supersymmetric and non-supersymmetric contexts, by which one can recover the spectrum of fermion masses through the localization of matter fields in an extra dimension. Orbifold boundary conditions allow us to complete the Arkani-Hamed-Schmaltz model for the first time, resulting in a theory which actually contains chiral matter. Going further, we construct two new non-supersymmetric models which successfully realize quark and lepton flavor from an underlying theory containing only parameters of order one. We have examined the constraints from flavor-changing neutral currents and CP violation as applied to the Kaon system, and find that our new models relax the experimental bounds on the fundamental scale compared to those on the AS model.

However, these constraints remain strong, requiring $M_* \gtrsim 10^3 M_W$, and disfavor the use of large extra dimensions to explain both flavor and the hierarchy problem. Thus we consider supersymmetric theories, where we can use much smaller extra dimensions, safe from flavor constraints related to KK modes of the gauge bosons. After reviewing the powerful notation that expresses 5 dimensional supersymmetry in terms of superfields, we find non-trivial flat directions which preserve $\mathcal{N} = 1$ supersymmetry and can themselves be used to localize chiral superfields.

These tools allow us to supersymmetrize our most successful non-supersymmetric flavor model, by invoking an orbifold and odd mass terms for hypermultiplets which result in chiral fields localized around the orbifold fixed points. One possible origin of this odd mass could be from the $\mathcal{N} = 2$ superpartners of the 5d gauge superfield for a U(1) gauge group which is maximally broken. Another interesting supersymmetric model is compactified without the orbifold, and generates a chiral theory by projecting chiral brane fields into the bulk through mixing with bulk hypermultiplets. This allows us to consider a supersymmetric version of the AS model, where the Higgses (as members of hypermultiplets) as well as the fermions are localized across the extra dimension.

If supersymmetry-breaking is also extra-dimensional, as in gaugino-mediation, the fact that the extra dimension plays a dual role can manifest itself in the low energy superpartner mass spectrum, and allows one to see evidence for the mechanism of flavor by carefully measuring the superparticle masses. If instead supersymmetry is broken by an exponentially small twist in the boundary conditions (i.e., the Scherk-Schwarz mechanism), our model successfully realizes a field theory mechanism for small Yukawas without disrupting the flavor degeneracy of the sfermions. Unfortunately, we do not know of any new weak-scale predictions in this case.

To summarize, an extra dimension allows for a new perspective on many of the puzzles in particle physics today. In assembling specific tools for one exciting feature — the localization of fields — it is our hope that these will prove useful in building models that are on the one hand beautiful and elegant, and on the other complete and realistic.

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